Solve System of Initial Value ODEs Using Fourth Order Runge Kutta Method

Objectives

- Solve a System of First order ODE using 4th Order Runge – Kutta Method
- A second or higher order ODE can be converted to a system of first order ODEs and can be solved using this procedure.
- The methods for solving a single first-order ODE can be used to solve systems of coupled first order ODEs.

The Fourth – Order Runge Kutta Method

- Consider a general non – linear first order ODE of the form
- $y' = f(t, y)$, $y(t_0) = y_0$
- $y_{n+1} = y_n + \left(\frac{1}{6}\right) * (\Delta y_1 + 2 * \Delta y_2 + 2 * \Delta y_3 + \Delta y_4)$
- $\Delta y_1 = h * f(t_n, y_n)$
- $\Delta y_2 = h * f\left(t_n + \frac{h}{2}, y_n + \frac{\Delta y_1}{2}\right)$
- $\Delta y_3 = h * f\left(t_n + \frac{h}{2}, y_n + \frac{\Delta y_2}{2}\right)$
- $\Delta y_4 = h * f(t_n + h, y_n + \Delta y_3)$

The Fourth – Order Runge Kutta Method

- Example ODE Problem
- $\frac{dy1}{dx}$ = - 0.5 * y1
- $\frac{dy2}{dx}$ = 4 - 0.3 * y2 – 0.1 * y1
- From x = 0 to 2 with a step size of 0.5;
- The initial condition at *x* = 0 is *y1* = 4, y2 = 6.
- Employ Procedure similar to the one used for solving the system of ODEs using Explicit Euler Method.

Summary

In this video,

- We presented the Fourth Order Runge Kutta method to solve a system of Initial Value ODEs

- The Fourth Order Runge Kutta method is conditionally stable.

- The global error is $0(\Delta t^4)$.

- In the next video, we can look at methods to solve Stiff ODEs